Melosh Construction Of Relativistic Three-Quark Baryon Wave Functions

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Abstract

A method for constructing a **complete set** of relativistic three-quark states in light front dynamics is implemented for the nucleon, N*(1520) and N*(1535). This approach facilitates constructing states containing virtual antiquarks and a physical interpretation is provided in terms of transition amplitudes from quark to quark-gluon or quark-Goldstone boson Fock states of chiral dynamics generated by flux tube breaking expected in QCD at intermediate distances.

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I. INTRODUCTION

In light front dynamics [1] (LF) and in Dirac's point form of relativistic few-body physics [2], [3], hadron wave functions may be boosted kinematically, that is, independent of interactions. Because form factors depend on boosted wave functions this attractive feature has motivated many recent electroweak form factor calculations [4].

Because there is no spontaneous creation of massive fermions in LF dynamics that is quantized on the null plane, the quantized vacuum is fairly simple, so that a constituent Fock space expansion is practical where partons are directly related to the hadron.

Here we construct relativistic baryon basis states in light front dynamics, although all important results are also valid in the point form, where four-velocities are replacing the light cone momentum variables. The null plane is invariant under seven of the ten Poincaré generators. A subgroup of six of these generators form the stability group that acts transitively on states of a given mass-shell hyperboloid. As a result, the total momentum separates from the internal momentum variables [5]. This is one of the reasons why LF dynamics is usually formulated in momentum space. Thus, wave functions depend only on the relative momentum variables and, being invariant under kinematic Poincaré transformations, are completely determined once they are known at rest.

Except for Sect. III we deal with nucleon basis states. Except for Sect. V, we have applications in mind to spacelike electroweak form factors in the Breit frame (with momentum transfer $q^+ = q^0 + q^3 = 0$) of LF dynamics excluding the complications of timelike exclusive processes (which can be treated [6]). As a rule, applications involve truncating the Fock space expansion by particle number, which does not violate Lorentz invariance because in LF dynamics the boost operators are kinematic, that is, do not contain interactions.

Three-quark wave functions for the nucleon have been constructed in the constituent quark model as products of a totally symmetric momentum and a nonstatic spin-flavor wave function which is an eigenstate of the spin and total angular momentum (squared) and its projection on the light cone axis. When nonstatic spin-flavor wave functions are built from the nonrelativistic quark model (NQM) via Melosh rotations [7], (see ref. [8] for a review and refs.) they form a Hilbert space of relativistic three-quark states that we shall call the Pauli-Melosh basis. It is in one-to-one correspondence with the NQM and manifestly orthogonal.

An alternative effective field theory approach starts from three-quark nucleon interaction Lagrangians [9] in light front dynamics which provide the nonstatic spin-flavor wave functions in a Dirac matrix representation. When the triangle Feynman diagram for a form factor is projected to the null-plane the radial momentum wave functions are defined in terms of three-quark-nucleon vertex functions and a totally symmetric energy denominator (three-quark propagator). Based on Lorentz covariance, parity and SU(2) isospin invariance there are eight independent couplings and three totally symmetric couplings. This Dirac basis is manifestly Lorentz covariant under kinematic transformations.

Here we wish to address the problem in that there are significantly more relativistic hadron wave functions in the Dirac basis than compared to the Pauli-Melosh basis. For example, there are three nucleon states compared to the single S-state of the NQM in the static limit and five $N^*(1535)$ states.

The alternative Bargmann-Wigner basis [10] (BW), which is in one-to-one correspon-

dence with the Dirac basis [8], sheds light on this problem from another point of view. It is based on the equivalence of the infinite momentum frame (IMF) and light front dynamics which implies that quarks bound in a hadron in the IMF are all collinear, that is, have equal velocities $p_i/m = P/M$, where M is the baryon mass and m a constituent quark mass. The BW basis constructs three-quark baryon Fock states in terms of baryon Dirac spinors (U, V). There is only one totally symmetric nucleon basis state (UUU). It corresponds to the nucleon ground state of the Pauli-Melosh basis. The other two totally symmetric three-quark states of the BW and Dirac bases contain two Dirac V-spinors and one U-spinor.

In this paper it is shown that a set of spin-rotated spinors are required to account for all three-quark-nucleon couplings, that is, all three non-static spin-flavor wave functions of the nucleon. Corresponding results are valid for N*s, the baryon octet and other baryons.

The paper is organized as follows. In Sect. II we introduce spin-rotated spinors based on the complete U, V basis, which are used in Sect. IV to construct the complete basis of three-quark spin-flavor wave functions in LF dynamics. In Sect. III we provide the transformation between the BW and Dirac representations of the three-quark couplings for the nucleon, N*(1520) and N*(1535) in a transparent approach that avoids overlap matrix elements of both bases used in ref [8]. After symmetrizing we find three and five spin-flavor components for nucleon and N*(1520), N*(1535), respectively. In Sect. IV we start from the instant form to construct the complete set of relativistic spin-flavor wave functions for positive-energy quarks. In Sect. V we consider the possibility of v-spinors in the wave functions and discuss their possible origin in light front dynamics. We discuss a physical interpretation of baryon states with v spinors as part of transition amplitudes from three-quark states to quark-gluon or quark-Goldstone boson Fock states via flux-tube breaking at intermediate distances in QCD.

II. SPIN-ROTATED LIGHT-CONE SPINORS

When the bound quarks of the nucleon are in the infinite momentum frame (IMF), or equivalently in the null plane of light front dynamics (LF), they are all collinear [11]

$$\frac{p_i}{m_i} = \frac{P}{M}, \text{ as P} \to \infty. \tag{1}$$

The transformation to the IMF amounts to a change of the usual momentum variables [12]

$$p^\mu = (p^0, \mathbf{p}) \text{ to } (\mathrm{p}^+ = \mathrm{p}^0 + \mathrm{p}^3,\ \mathbf{p}_\perp = (\mathrm{p}^1, \mathrm{p}^2),\ \mathrm{p}^- = \mathrm{p}^0 - \mathrm{p}^3) \tag{2}$$

to light cone momentum components. As in any Hamiltonian version of field theory, partons are on their mass shell so that the light cone energy $p^- = (m^2 + \mathbf{p}_\perp^2)/p^+ > 0$, in contrast to the square root ambiguity for p^0 in the instant form. In transitions the p^- variable is not conserved.

The usual instant-form quark spinors [13] $u_\lambda^{inst}(k)$, $v_\lambda^{inst}(k)$ under this transformation to the IMF will change their form and become light-cone spinors $u_\lambda^{LC}(k)$, $v_\lambda^{LC}(k)$ denoted by a superscript LC (see ref. [8] for a review) and λ is the helicity. Note that our v-spinors obey $v_\lambda(k) = \gamma_5 u_\lambda(k)$ in contrast to $v_\lambda = C\bar{u}_\lambda^T$ of ref. [13].

The total momentum spinors $U_\uparrow$, $U_\downarrow$ of the nucleon satisfy the free Dirac equation

$$(\gamma \cdot P - M)U_\lambda = 0, \tag{3}$$

where M is the nucleon mass, P its total momentum. In the rest frame of the nucleon the total momentum spinors have the form $U_\uparrow^T = (1,0,0,0),\ U_\downarrow^T = (0,1,0,0)$. The corresponding $V_\uparrow,\ V_\downarrow$ spinors satisfy

$$(\gamma \cdot P + M)V_\lambda = 0, \tag{4}$$

and have the rest-frame forms $V_\uparrow^T = (0,0,1,0),\ V_\downarrow = (0,0,0,1)$. The $U,\ V$ spinors are the building blocks of the Bargmann-Wigner (BW) basis.

The unitary transformation between instant-form and light front spinors is given by the Melosh rotation matrices [14]

$$u_\lambda^{inst} = \sum_\xi \mathcal{R}^{(1)}_{\lambda\xi} u_\xi^{LC}, \quad v_\lambda^{inst} = \sum_\xi \mathcal{R}^{(2)}_{\lambda\xi} v_\xi^{LC}, \tag{5}$$

where λ and ξ represent the helicities and

$$\mathcal{R}^{(1)}_{\lambda\xi} = \mathcal{R}^{(2)}_{\lambda\xi} = \frac{1}{\sqrt{2k^+(m+k^0)}} \begin{pmatrix} k^+ + m & -k^R \\ k^L & k^+ + m \end{pmatrix}_{\lambda\xi}. \tag{6}$$

Here $k^{R,L} = k^1 \pm ik^2$ are conventional abbreviations for the quark momentum in the nucleon rest frame and m is the quark mass. The Melosh matrix for a single quark can also be written as an overlap of a quark spinor and the nucleon spinors $U_\lambda,\ V_\lambda$,

$$\bar{u}_\xi^{LC} U_\lambda = \frac{1}{2\sqrt{mk^+}} \begin{pmatrix} k^+ + m & -k^R \\ k^L & k^+ + m \end{pmatrix}_{\lambda\xi} \tag{7}$$

$$\bar{v}_\xi^{LC} V_\lambda = -\frac{1}{2\sqrt{mk^+}} \begin{pmatrix} k^+ + m & -k^R \\ k^L & k^+ + m \end{pmatrix}_{\lambda\xi} \tag{8}$$

in the nucleon rest frame, so that the Melosh rotations may be written in the shorter form

$$\mathcal{R}^{(1)}_{\lambda\xi} = N\bar{u}_\xi^{LC} U_\lambda = \mathcal{R}^{(2)}_{\lambda\xi} = -N\bar{v}_\xi^{LC} V_\lambda, \tag{9}$$

with normalization $N = \sqrt{\frac{2m}{m+k^0}}$. In any frame, therefore,

$$\begin{pmatrix} \bar{u}_\uparrow^{LC} U_\uparrow & \bar{u}_\downarrow^{LC} U_\uparrow \\ \bar{u}_\uparrow^{LC} U_\downarrow & \bar{u}_\downarrow^{LC} U_\downarrow \end{pmatrix} = \frac{1}{2\sqrt{mMp^+P^+}} \begin{pmatrix} Mp^+ + mP^+ & P^R p^+ - P^+ p^R \\ P^+ p^L - P^L p^+ & Mp^+ + mP^+ \end{pmatrix}. \tag{10}$$

In a different notation these Melosh rotations are considered in [14].

The block diagonal form of the transformations, Eq. 9, motivates us to define off-diagonal transformations

$$w_\lambda^{inst} = \sum_l \left(\bar{v}_l^{LC} U_\lambda\right) u_l^{LC}, \quad z_\lambda^{inst} = \sum_l \left(\bar{u}_l^{LC} V_\lambda\right) v_l^{LC}, \tag{11}$$

so that

$$\bar{v}_\xi^{LC} U_\lambda = \frac{1}{2\sqrt{mp^+}} \begin{pmatrix} k^+ - m & k^R \\ k^L & -k^+ + m \end{pmatrix}_{\lambda\xi} \equiv \frac{1}{N'} \mathcal{R}^{(3)}_{\lambda\xi} \tag{12}$$

$$\bar{u}_\xi^{LC} V_\lambda = -\frac{1}{2\sqrt{mp^+}} \begin{pmatrix} k^+ - m & k^R \\ k^L & -k^+ + m \end{pmatrix}_{\lambda\xi} \equiv -\frac{1}{N'} \mathcal{R}^{(4)}_{\lambda\xi}, \tag{13}$$

with normalization $N' \equiv \sqrt{\frac{2m}{k^0 - m}}$. These transformations are manifestly unitary and relate

$$w_\lambda^{inst} = \sum_\xi \mathcal{R}^{(3)}_{\lambda\xi} v_\xi^{LC}, \quad z_\lambda^{inst} = \sum_\xi \mathcal{R}^{(4)}_{\lambda\xi} u_\xi^{LC}. \tag{14}$$

The z_λ^{inst}, w_λ^{inst} spinors obey the Dirac equations

$$(\gamma \cdot k - m) z_\lambda^{inst}(k) = 0, \quad (\gamma \cdot k + m) w_\lambda^{inst}(k) = 0. \tag{15}$$

We can see that the w^{inst} and z^{inst} spinors differ from u^{inst} and v^{inst} spinors only in replacing $k^0 + m$ by $k^0 - m$. Thus in highly relativistic cases, $k^0 + m \sim k^0 - m$ and $w_\lambda^{inst} \to u_\lambda^{inst}$, $z_\lambda^{inst} \to v_\lambda^{inst}$. In the static limit, however, the z^{inst}, w^{inst} spinors appear to diverge, but on closer inspection the static limit is finite yet depends on the direction in which it is taken. Let us approach the static limit so that

$$k^x = k\alpha,\ k^y = k\beta,\ k^z = k\gamma, \quad \alpha^2 + \beta^2 + \gamma^2 = 1, \ \text{ as k} \to 0. \tag{16}$$

Then

$$\sqrt{k^0 - m} \sim \frac{k}{\sqrt{2m}},\ \frac{k^z}{\sqrt{k^0 - m}} \sim \gamma\sqrt{2m},\ \frac{k^R}{\sqrt{k^0 - m}} \sim (\alpha + i\beta)\sqrt{2m} \text{ as k} \to 0, \tag{17}$$

and

$$(z_\uparrow^{inst})^T = (\gamma,\ \alpha + i\beta,\ 0,\ 0)$$

remains finite and normalized to unity, but clearly depends on how one approaches the static limit. Similar results hold for the other three spinors.

In fact, the w^{inst}, z^{inst} spinors are directly related to the u^{inst}, v^{inst} spinors by the spin rotation

$$w_\lambda^{inst}(k) = \vec{\sigma} \cdot \hat{k} v_\lambda^{inst}(k), \quad z_\lambda^{inst}(k) = \vec{\sigma} \cdot \hat{k} u_\lambda^{inst}(k), \tag{18}$$

where the unit vector $\hat{k} = \vec{k}/k$, $k = \sqrt{\vec{k}^2}$. This spin rotation is clearly not well defined for $k \to 0$. To prove this relation for z^{inst}, for example, we repeatedly substitute

$$\vec{k}^2 = (k^0)^2 - m^2 = (k^0 - m)(k^0 + m) \tag{19}$$

in

$$\begin{aligned} \vec{\sigma} \cdot \hat{k} u^{inst}(k) &= \sqrt{\frac{k^0 + m}{2m}} \begin{pmatrix} \vec{\sigma} \cdot \hat{k} \\ \frac{k}{k^0 + m} \end{pmatrix} \chi = \frac{k}{\sqrt{2m(k^0 + m)}} \begin{pmatrix} \frac{k^0 + m}{k^2} \vec{\sigma} \cdot \vec{k} \\ 1 \end{pmatrix} \chi \\ &= \sqrt{\frac{k^0 - m}{2m}} \begin{pmatrix} \frac{\vec{\sigma} \cdot \vec{k}}{k^0 - m} \\ 1 \end{pmatrix} \chi = z^{inst}. \end{aligned} \tag{20}$$

The consistency of this ambiguity of the static limit with the nature of the UVV invariants is discussed in Sect. IV.

The reason for introducing these spin rotated w and z spinors and their usefulness become transparent for multi-quark states only. But let us emphasize here already that we may view the matrix elements involved in Eq. 11 as unitary transformations from light cone to instant spinors via the BW (or IMF) baryon basis states $U,\ V$. We shall show that, for the resulting **three-quark** basis to be complete, one needs to go via V states as well as U states. In this sense Eq. 11 provides the missing off-diagonal elements from U_λ to $v_{\lambda'}$ and V_λ to $u_{\lambda'}$.

In Section IV we shall use these Melosh rotations for w and z states to construct all three-quark states of the Dirac and BW representations, which will make their equivalence manifest.

III. CONVERSION OF DIRAC TO BARGMANN-WIGNER BASIS

In this section we review the connection between two forms of nucleon wave functions, one in the Dirac basis and the other in the Bargmann-Wigner basis. We develop a simple and transparent method to establish the one-to-one correspondence between the Dirac and Bargmann-Wigner bases that avoids the evaluation of overlap matrix elements used in ref. [8].

When the nucleon is treated as a spin-$\frac{1}{2}$ field, it is a third-rank spinor $\Psi_{[\alpha\beta]\gamma}$ which satisfies the free Dirac equation for each spinor index

$$(\gamma\cdot P - M)_\alpha^{\alpha'}\Psi_{[\alpha'\beta]\gamma} = (\gamma\cdot P - M)_\beta^{\beta'}\Psi_{[\alpha\beta']\gamma} = (\gamma\cdot P - M)_\gamma^{\gamma'}\Psi_{[\alpha\beta]\gamma'} = 0. \tag{21}$$

These constraints and the total symmetry under permutation of the three spinor indices lead to the spinor form $[(\gamma\cdot P + M)\gamma_5 C]_{\alpha\beta}U_\lambda(P)$ of $\Psi_{[\alpha\beta]\gamma}$ [10], which is antisymmetric under the exchange of the α,β indices. Here $C = i\gamma^2\gamma^0$ is the charge conjugation matrix.

However, in QCD and in quark models in particular the nucleon is no longer a local field. Various bound state nucleon wave function components may be related to different three-quark-nucleon vertices defined by corresponding interaction Lagrangians. Such an approach has recently [9] been adopted for a null-plane projection of the Feynman triangle diagram for spacelike electroweak form factors of the nucleon. From Lorentz invariance and symmetries under permutations of three quarks, a basis of Dirac γ-matrix representations has been constructed for the spin-flavor components of these couplings (see ref. [8] for a review) which is equivalent to the BW-basis. The specific spin coupling listed above reduces to the nonrelativistic quark model S-state in the static limit. This spin wave function dominates the applications of relativistic quark models.

The wave function of a proton can be written in the uds basis as

$$\begin{aligned} |P_\uparrow\rangle \ &= \ |\mathrm{mom.}\rangle \otimes [\tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow \otimes \tfrac{1}{\sqrt{2}}(\mathrm{ud}-\mathrm{du})\mathrm{u} \\ &+ \ (\mathrm{MS})_{\mathrm{spin}} \otimes (\mathrm{MS})_{\mathrm{isospin}}] \otimes |\mathrm{SU}(3)_{\mathrm{color}}\rangle, \end{aligned} \tag{22}$$

where MS stands for the mixed symmetric combination.

In order to convert the spin structure of this wave function to Lorentz covariant form, we start rewriting two-particle spin wave functions in terms of Pauli matrices. We know that $|\uparrow\rangle\ |\downarrow\rangle - |\downarrow\rangle\ |\uparrow\rangle$ is the antisymmetric combination of two spin-$\frac{1}{2}$ particles. If we recognize that $|\uparrow\rangle \to \binom{1}{0},\ \ |\downarrow\rangle \to \binom{0}{1}$ and carry out the direct product, then we get

$$\left(\begin{pmatrix}1\\0\end{pmatrix}(0 \quad 1) - \begin{pmatrix}0\\1\end{pmatrix}(1 \quad 0)\right) = \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} = -i\sigma_2, \tag{23}$$

that is, the off-diagonal matrix σ_2 represents antisymmetric coupling. This identity has a similar form when we go to the four-dimensional Dirac space, where the basis is comprised of four-components Dirac spinors $U_\uparrow, U_\downarrow, V_\uparrow, V_\downarrow$ of the nucleon from the previous Sect.II. For this case the decomposition takes the form

$$\Gamma^{\alpha\beta} = (U_\uparrow^\alpha U_\downarrow^\alpha V_\uparrow^\alpha V_\downarrow^\alpha)_\mu \Gamma^{\mu\nu} \begin{pmatrix} U_\uparrow^\beta \\ U_\downarrow^\beta \\ V_\uparrow^\beta \\ V_\downarrow^\beta \end{pmatrix}_\nu , \tag{24}$$

where Γ is one of the sixteen Dirac matrices

$$\begin{gathered} \gamma^\mu C, \sigma^{\mu\nu} C \ (1 \leftrightarrow 2 \text{ symmetric}), \\ \gamma^\mu \gamma_5 C, \gamma_5 C, C \ (1 \leftrightarrow 2 \text{ antisymmetric}). \end{gathered} \tag{25}$$

The entries for the Γ matrices have a twofold meaning: one as the coupling of the U,V spinors with different helicities and the other as the direct product of the spinor elements.

The above identity (24) is most easily checked in the nucleon rest frame. To see how to convert the G_i of the Dirac basis in Table I into U, V spinors we proceed in two steps. First we decompose the UV coupling and then the spin structure. For example, for $G_2 \sim \gamma_5 C \otimes U^\uparrow$ because

$$\begin{aligned} \gamma_5 C &= \gamma_5 i\gamma^2\gamma^0 = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}^{UV} \otimes \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}^{spin} \\ &= (U \quad V)\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}U\\V\end{pmatrix} \otimes (\uparrow \quad \downarrow)\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}\uparrow\\\downarrow\end{pmatrix} \\ &= (UU + VV) \otimes (-\uparrow\downarrow + \downarrow\uparrow). \end{aligned} \tag{26}$$

Including the third quark, we obtain

$$\begin{aligned} G_2 = \gamma_5 C U_\uparrow &= (UU + VV) \otimes (-\uparrow\downarrow + \downarrow\uparrow) \otimes U^\uparrow \\ &= -U^\uparrow U^\downarrow U^\uparrow + U^\downarrow U^\uparrow U^\uparrow - V^\uparrow V^\downarrow U^\uparrow + V^\downarrow V^\uparrow U^\uparrow. \end{aligned} \tag{27}$$

The conversion to the BW basis for all the eight invariants in Table I proceeds in this way, and they are displayed in Table II.

The spin part of the wave function is constructed by choosing two of the quarks coupling via $(\Gamma^1)_{\alpha\beta}$, abbreviated as (α, β), with Γ^1 from Eqs. 24, 25 having definite permutation symmetry with respect to the exchange of the two spinor indices. The third quark index is added by combining a Dirac matrix Γ^2, which is one of the basic sixteen 1, γ_5, γ^μ,$\gamma_5\gamma^\mu$, $\sigma_{\mu\nu}$ with the nucleon spinor $u_N(P)$. Hence the total spinor components have the third-rank tensor structure

$$\Psi_{[\alpha\beta]\gamma} = (\Gamma^1)_{\alpha\beta}(\Gamma^2 u_N(P))_\gamma,$$

which may be evaluated for u- and v-spinors. In light front quark model applications, such as form factors defined by a triangle Feynman diagram, such spin couplings are sandwiched

between u-spinors only and called spin wave functions. Next the isospin coupling is multiplied in with matching (12) permutation symmetry. However, baryon wave functions exhibit the totally symmetric form (12)3+(23)1+(31)2, except for the color part, so that we need to symmetrize the spin invariants (12)3 that we have constructed so far.

To make a totally symmetric spin-flavor wave function (without the color indices), we combine the (12) symmetric isospin $\vec{\tau} \otimes \vec{\tau}\phi_N$ with the symmetric (12) spin coupling, where ϕ_N is the isospin wave function of the nucleon. Similarly for the (12) antisymmetric combination, the mixed antisymmetric isospin combination is given by $i\tau_2 \otimes \phi_N$. Thus for the distinguishable (12)3 quark system we have the eight independent spin-isospin basis states shown in Table I.

Once we have the spinor invariants for the distinguishable quarks (12)3, we need to permute them according to (12)3+(23)1+(31)2. To simplify the basis further, we calculate the isospin matrix elements explicitly using the *uds* basis, where in a proton, (12)3 denotes the quark configuration *uud*. Let us take G_1 as an example,

$$G_1 = M \sum [\phi_1^\dagger i\tau_2 \phi_2^\dagger \phi_3 \phi_N]_{isospin} \otimes [(C)_{12} \otimes (u_N)_3]_{spin}. \tag{28}$$

Under permutation,

$$\begin{aligned} S_1 = M \sum [\phi_1^\dagger i\tau_2 \phi_2 \phi_3^\dagger \phi_N]_{isospin} \otimes [(C)_{12} \otimes (u_N)_3]_{spin} + (23)1 + (31)2 \\ = [(23)1]_{spin} - [(31)2]_{spin}. \end{aligned} \tag{29}$$

Dropping the subscript *spin* in the above equation, the totally symmetric coupling S_1 corresponding to G_1 becomes

$$S_1 = (C)_{23}(u_N)_1 - (C)_{31}(u_N)_2. \tag{30}$$

Similarly for a (12) symmetric $\vec{\tau}_{12} \otimes \vec{\tau}_3$ coupling in isospin space we find the isospin matrix elements -2,1,1 for (12)3, (23)1 and (31)2 couplings, respectively. Then the symmetrized spinor invariant G_3 becomes

$$S_3 = -2M(\gamma^\mu C)_{12} \otimes (\gamma_5 \gamma_\mu u_N)_3 + (23)1 + (31)2. \tag{31}$$

In order to rearrange the (23)1 and (31)2 terms in (12)3 order, we use the Fierz rearrangement Table III (see also Table I of ref. [15]) to get

$$S_1 = (23)1 - (31)2 = \frac{1}{2}G_3 - \frac{1}{4}G_7. \tag{32}$$

The final results are shown in Table IV. From Table IV, we can find out that there exist only three independent components, because there are five linear relationships between them, namely,

$$\begin{aligned} S_1 &= S_2 - S_4, \\ S_3 &= 3S_4, \\ S_5 &= S_4 - S_6, \\ S_7 &= 12S_2 - 6S_4, \\ S_8 &= 3S_2 - 2S_4 + 2S_6. \end{aligned} \tag{33}$$

Thus, we can choose S_2, S_4, S_6 as the three independent couplings for the nucleon wave function.

The construction of spinor invariants for nucleon resonance states follows the same procdures except for replacing the total momentum spinor u_N by a Rarita-Schwinger spinor for spin-$\frac{3}{2}$

$$U_\lambda^{\frac{1}{2}^-,\mu} = \sum_{m_1,m_2,m_1+m_2=\lambda} C^{1\ \frac{1}{2}\ \frac{1}{2}}_{m_1 m_2\ \lambda}\ \hat{\epsilon}^\mu_{m_1} u_{m_2} \tag{34}$$

$$U_\lambda^{\frac{3}{2}^-,\mu} = \sum_{m_1,m_2,m_1+m_2=\lambda} C^{1\ \frac{1}{2}\ \frac{3}{2}}_{m_1 m_2\ \lambda}\ \hat{\epsilon}^\mu_{m_1} u_{m_2}, \tag{35}$$

where $u_m(P)$ is a Dirac spinor, and the four mutually orthogonal vectors P^μ, and the polarization vectors $\hat{\epsilon}^\mu_{\pm,0}$ together form basis of Minkowski space. The Rarita-Schwinger spinor has eight independent components because it satisfies the constraint $\gamma_\mu U_\lambda^{\frac{3}{2},\mu} = 0$, which shows that there is no spin-$\frac{1}{2}$ contribution. The orthogonality between the spinors is guaranteed by that of the Clebsch-Gordan coefficients. The contraction between the relative momentum variables s_μ and $U_{1/2}^{1/2,\mu}$ gives

$$s_\mu U_{\frac{1}{2}}^{\frac{1}{2},\mu} = -\sqrt{\frac{1}{3}}(s^z u_\uparrow + s^R u_\downarrow) = -\sqrt{\frac{1}{3}}(\gamma_5 \gamma \cdot s u_\uparrow). \tag{36}$$

This is consistent with the spinor invariants for $N^*(1535)$ constructed in reference [8], where the total momentum wave function was written as $s_\mu u_\lambda$ and contracted with γ^μ instead of $(s_\mu U_{\frac{1}{2}}^{\frac{1}{2},\mu})$ in our case.

For $N^*(1535)$, $N^*(1520)$ the invariants are constructed by replacing the Dirac spinor by $\gamma_5(s_\mu U_\lambda^{(\frac{3}{2},\frac{1}{2}),\mu})$ and are listed in Tables V and VI. The extra γ_5 is necessary for the correct parity. The symmetrization among the three quarks follows the same procedure as that for the nucleon and the results are listed in Table VII for the Dirac basis.

Because of the presence of the relative momentum s_μ ($s_3 = p_1 - p_2$, etc.) the number of linear relations between the eight symmetrized states is reduced to three, namely

$$\begin{aligned} 2(S_1' + S_2') - S_7' &= 0, \\ S_2' + S_6' - S_3' + S_5' - S_8' &= 0, \\ 4S_1' + 3S_3' - S_4' - S_7' &= 0. \end{aligned} \tag{37}$$

This means that there are now five independent spinor components for $N^*(1535)$, $N^*(1520)$ each.

To summarize what we have accomplished so far, the eight independent relativistic couplings can be transformed into equivalent linear combinations of direct products of three Dirac spinors, which can be either $U_\lambda(P)$, the positive energy spinors or $V_\lambda(P)$, the negative energy spinors, where λ means helicity. This is what is called Bargmann-Wigner basis in ref. [8] as compared to the Dirac matrix representation or basis. While the invariants G_i of the Dirac basis can be easily interpretated as couplings between the three quarks and the nucleon, the invariants expressed in the BW basis have the advantage of being more easily permuted. Moreover, in Sect. IV, we shall use the BW representation as a means of transforming the spin-flavor wave functions from the instant to the LF form.

IV. THREE-QUARK NUCLEON STATES

With $q^3 - N$ couplings written in both Dirac and U, V forms, we are ready to generate the nucleon wave functions being used in light front dynamics. For simplicity we consider only the spin structure of the nucleon, ignore its isospin and treat quarks as distinguishable.

The relativistic spinor basis is constructed from nonrelativistic Pauli spinors χ. If we focus on the (12) quark pair, then the singlet pair state

$$|0,0\rangle \equiv \sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, \chi_{\lambda_1} \otimes \chi_{\lambda_2} \tag{38}$$

the first step of the relativistic generalization is to replace Pauli spinors by Dirac spinors of positive energy in the instant form

$$|0,0\rangle^{(uu)} \equiv \sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, u^{inst}_{\lambda} \otimes u^{inst}_{\lambda_2}. \tag{39}$$

Upon transforming the instant to light-front spinors, we apply the first part of Eq. (5) to get

$$|0,0\rangle^{(uu)} = \sum_{\xi_1,\xi_2} \left[\sum_{\lambda_1,\lambda_2} \left(C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, \mathcal{R}^{(1)}_{\lambda_1\xi_1} \mathcal{R}^{(1)}_{\lambda_2\xi_2} \right) \right] u^{LC}_{\xi_1} \otimes u^{LC}_{\xi_2}. \tag{40}$$

Substituting Eq. (9) for $\mathcal{R}^{(1)}_{\lambda\xi}$ we obtain

$$|0,0\rangle^{(uu)} = \frac{2m}{m+k^0} \sum_{\xi_1,\xi_2} \left[\bar{u}^{LC}_{\xi_1} \Big(\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, U_{\lambda_1} \otimes U_{\lambda_2} \Big) (\bar{u}^{LC}_{\xi_2})^T \right] u^{LC}_{\xi_1} \otimes u^{LC}_{\xi_2}. \tag{41}$$

From the conversion of the Dirac basis to the BW-basis in the previous Section III, we know that in the nucleon rest frame

$$\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, U_{\lambda_1} \otimes U_{\lambda_2} = \frac{1+\gamma_0}{2\sqrt{2}} \gamma_5 C \tag{42}$$

is valid. Therefore

$$|0,0\rangle^{(uu)} = \frac{2m}{\sqrt{2}(m+k^0)} \sum_{\xi_1,\xi_2} \left[\bar{u}^{LC}_{\xi_1} \frac{1+\gamma_0}{2} \gamma_5 C (\bar{u}^{LC}_{\xi_2})^T \right] u^{LC}_{\xi_1} \otimes u^{LC}_{\xi_2}. \tag{43}$$

This can be generalized to an arbitrary frame by boosting the nucleon to momentum P, so that we have

$$|0,0\rangle^{(uu)} = \frac{m\sqrt{2}}{m+k^0} \sum_{\xi_1,\xi_2} \left[\bar{u}^{LC}_{\xi_1} \frac{\gamma\cdot P + M}{2M} \gamma_5 C (\bar{u}^{LC}_{\xi_2})^T \right] u^{LC}_{\xi_1} \otimes u^{LC}_{\xi_2}. \tag{44}$$

Combining this quark pair state with the third quark we get precisely the three-quark state that is usually considered in applications of relativistic quark models.

Now we are ready to extend the spin coupling method to quark pairs constructed via V-spinors, that is, there are similar new components starting from z spinors in the instant form,

$$\begin{aligned}|0,0\rangle^{(zz)} &\equiv \sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, z^{inst}_{\lambda} \otimes z^{inst}_{\lambda_2} \\ &= \sum_{\xi_1,\xi_2}\left[\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, \mathcal{R}^{(4)}_{\lambda_1\xi_1}\mathcal{R}^{(4)}_{\lambda_2\xi_2}\right]\ u^{LC}_{\xi_1}\otimes u^{LC}_{\xi_2} \\ &= -\tfrac{2m}{k^0-m}\sum_{\xi_1,\xi_2}\left[\bar{u}^{LC}_{\xi_1}(\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, V_{\lambda_1}\otimes V_{\lambda_2})(\bar{u}^{LC}_{\xi_2})^T\right]\ u^{LC}_{\xi_1}\otimes u^{LC}_{\xi_2}.\end{aligned} \tag{45}$$

From Eq. (45) and Eq. (41) we can form the linear combination

$$\frac{\sqrt{2}}{N^2}|0,0>^{(uu)} - \frac{\sqrt{2}}{N'^2}|0,0>^{(zz)} = -\sum_{\xi_1,\xi_2}[\bar{u}^{LC}_{\xi_1}\gamma_5 C(\bar{u}^{LC}_{\xi_2})^T]\ u^{LC}_{\xi_1}\otimes u^{LC}_{\xi_2}, \tag{46}$$

where in the next to last line we recall (Eq. 27)

$$\begin{aligned}-\tfrac{1}{\sqrt{2}}\gamma_5 C &= \tfrac{1}{\sqrt{2}}(U_\uparrow U_\downarrow - U_\downarrow U_\uparrow + V_\uparrow V_\downarrow - V_\downarrow V_\uparrow) \\ &= \sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0} U_{\lambda_1}U_{\lambda_2} + \sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0} V_{\lambda_1}V_{\lambda_2}.\end{aligned} \tag{47}$$

Thus, from the linear combination of $|0,0\rangle^{(uu)}$ and $|0,0\rangle^{(zz)}$ we can construct the spinor invariant G_2 with positive spinors u_1 and u_2 in the Dirac representation. This means that, if we are to account for all eight invariants G_i in Table I as independent spinor components with the positive energy spinors as in light-front dynamics, then it is necessary to include the z^{inst} spinors.

With almost the same steps we find

$$\frac{\sqrt{2}}{N^2}|0,0\rangle^{(uu)} + \frac{\sqrt{2}}{N'^2}|0,0\rangle^{(zz)} = -\sum_{\xi_1,\xi_2}[\bar{u}^{LC}_{\xi_1}\gamma_0\gamma_5 C(\bar{u}^{LC}_{\xi_2})^T]\ u^{LC}_{\xi_1}\otimes u^{LC}_{\xi_2}, \tag{48}$$

which is the quark pair part of G_6 in the rest frame of the nucleon. There are similar expressions for all other spin invariants.

We are now ready to complete the construction of relativistic, mixed-antisymmetric (MA) positive-energy three-quark spinor states.

From Eq. (11), we see that the U and V spinors come in when we transform the instant spinors to the light-front form. If we require the quark spinors to be of positive energy only, then there are only two ways to generate the u^{LC}_i

$$u^{inst}_\lambda = N\sum_\xi(\bar{u}^{LC}_\xi U_\lambda)u^{LC}_\lambda, \quad z^{inst}_\lambda = -N'\sum_\xi(\bar{u}^{LC}_\xi V_\lambda)u^{LC}_\lambda. \tag{49}$$

Since in the UV construction, the only allowed combinations are either three Us or two Vs and one U for parity reasons, in the instant form the quark spinors must be one of the following four combinations

$$\begin{aligned}UUU\ :\ |MA,\lambda\rangle^{(uuu)} &\equiv \sum_{\lambda_3,\lambda} C^{0\ \frac{1}{2}\ \frac{1}{2}}_{0\ \lambda_3\ \lambda}(\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, u^{inst}_{\lambda_1}\otimes u^{inst}_{\lambda_2})\otimes u^{inst}_{\lambda_3} \\ VVU\ :\ |MA,\lambda\rangle^{(zzu)} &\equiv \sum_{\lambda_3,\lambda} C^{0\ \frac{1}{2}\ \frac{1}{2}}_{0\ \lambda_3\ \lambda}(\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, z^{inst}_{\lambda_1}\otimes z^{inst}_{\lambda_2})\otimes u^{inst}_{\lambda_3} \\ |MA,\lambda\rangle^{(zuz)} &\equiv \sum_{\lambda_3,\lambda} C^{0\ \frac{1}{2}\ \frac{1}{2}}_{0\ \lambda_3\ \lambda}(\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, z^{inst}_{\lambda_1}\otimes u^{inst}_{\lambda_2})\otimes z^{inst}_{\lambda_3} \\ |MA,\lambda\rangle^{(zzu)} &\equiv \sum_{\lambda_3,\lambda} C^{0\ \frac{1}{2}\ \frac{1}{2}}_{0\ \lambda_3\ \lambda}(\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, u^{inst}_{\lambda_1}\otimes z^{inst}_{\lambda_2})\otimes z^{inst}_{\lambda_3}.\end{aligned} \tag{50}$$

This gives a total of four states. We know their linear independence from their Dirac basis representations. It can be shown that linear combinations of states constructed from the above four states can generate G_1, G_2, G_4 and G_6. And similarly starting from the mixed-symmetric combination we can generate the remaining four G_i couplings. These results display the explicit separation of the Dirac matrix forms of three-quark-nucleon couplings into states whose Dirac spinor content and spin couplings are manifest. In ref. [9] it is shown that different quark-nucleon wave functions characterized by various G_i have important physical consequences. For example, the nonrelativistic quark model (NQM) gives the same fall-off with increasing momentum transfer for electric (G_E^p) and magnetic (G_M^p) form factors of the proton. The Melosh rotated ground state wave function of the NQM is given by $G_2 + G_6$. However, wave functions of G_2, G_6 and $G_2 + G_6$ type generate different slopes of G_E^p/G_M^p as functions of momentum transfer [9], so that G_2 is in better agreement with recent data from Jefferson Laboratory [16] than $G_2 + G_6$, or G_6. For the neutron charge form factor the agreement of G_2 with the data and disagreement of $G_2 + G_6$ and G_6 are even more pronounced.

The UVV invariants differ from UUU in that they can not be obtained from a well defined static limit via Melosh rotation. This explains why the z, w spinors do not have a well defined static limit which would circumvent this no-go theorem. In other words, the undefined static limit of the z, w spinors is the price one pays for the canonical Melosh construction of **all** spin-flavor couplings of the Dirac basis.

V. THREE-QUARK STATES WITH VIRTUAL ANTIQUARKS

In a typical electromagnetic form factor calculation the one-body quark current is sandwiched between three-quark wave functions of the nucleon [4] that contain only u-quarks (see the $uuu - N$ vertex of Fig.1a). No intermediate v-quarks occur in the triangle Feynman diagram because the quark propagator contains only a u-spinor piece with energy denominator $p^- - \frac{\mathbf{p}_\perp^2 + m^2}{p^+}$ in addition to the instantaneous part $\frac{\gamma^+}{2p^+}$, and any v-spinor coupled to the virtual photon is eliminated in the Breit frame (by $q^+ = 0$). Nonetheless, the three-quark-nucleon and -baryon couplings may occur in Feynman diagrams sandwiched between u- and v-spinors. In this context, we expect that v-states, and the $vuu - N$ vertex of Fig.1b in particular, are generated by flux-tube breaking in QCD. In fact, in ref. [17] the phenomenologically successful ${}^3\mathrm{P}_0$ quark-pair creation model of hadron decays is generalized to the (color electric) flux tube breaking mechanism expected to occur in QCD at intermediate distances. Because it is conceptually simpler we keep the approximate ${}^3\mathrm{P}_0$ quark pair creation vertex with its characteristic spin coupling [18] $\sim \bar{u}(\mathbf{p})v(-\mathbf{p}) \sim \sigma \cdot \mathbf{p}$ in our discussion. The spin matrix elements (in spherical basis) are the Clebsch-Gordan coefficients that couple the quark- antiquark spins to the triplet state. In hadronic decays the ${}^3\mathrm{P}_0$ vertex introduces the v-spinor that converts a three-quark spin-flavor invariant of uuu type to vuu type.

Now we are ready to extend the spin coupling method to quark pairs containing v-spinors and lift the restriction to positive energy spinors. Including two v^{inst} spinors to maintain positive parity, and in analogy with Eq. (39) we have

$$|0,0\rangle^{(vv)} \equiv \sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\, v^{inst}_{\lambda} \otimes v^{inst}_{\lambda_2}. \tag{51}$$

Applying the Melosh rotation $\mathcal{R}^{(2)}_{\lambda\xi}$ (Eq. 9), we obtain from Eq. 51

$$\begin{aligned}
|0,0\rangle^{(vv)} &= \sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0} \sum_{\xi_1,\xi_2} (\mathcal{R}^{(2)}_{\lambda_1\xi_1} v^{LC}_{\xi_1}) \otimes (\mathcal{R}^{(2)}_{\lambda_2\xi_2} v^{LC}_{\xi_2}) \\
&= -N^2 \sum_{\xi_1,\xi_2} \left[\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0} (\bar{v}^{LC}_{\xi_1} V_{\lambda_1})(\bar{v}^{LC}_{\xi_2} V_{\lambda_2})\right]\ v^{LC}_{\xi_1} \otimes v^{LC}_{\xi_2} \\
&= -\tfrac{2m}{m+k^0} \sum_{\xi_1,\xi_2} \left[\bar{v}^{LC}_{\xi_1} (\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0} V_{\lambda_1} \otimes V_{\lambda_2})(\bar{v}^{LC}_{\xi_2})^T\right]\ v^{LC}_{\xi_1} \otimes v^{LC}_{\xi_2}.
\end{aligned} \tag{52}$$

From the conversion in Section III, we recall that

$$\begin{aligned}
-\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0} V_{\lambda_1} V_{\lambda_2} &= \sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0} U_{\lambda_1} U_{\lambda_2} - \tfrac{1}{\sqrt{2}} \gamma_5 C \\
&= \tfrac{1}{\sqrt{2}} (\tfrac{1+\gamma_0}{2} \gamma_5 C - \gamma_5 C) = \tfrac{1}{\sqrt{2}} (\tfrac{\gamma_0 - 1}{2} \gamma_5 C).
\end{aligned} \tag{53}$$

Substituting Eq. (53) into Eq. (52) we get

$$|0,0\rangle^{(vv)} = \frac{m\sqrt{2}}{m+k^0} \sum_{\xi_1,\xi_2} \left[\bar{v}^{LC}_{\xi_1} \frac{\gamma_0 - 1}{2} \gamma_5 C (\bar{v}^{LC}_{\xi_2})^T\right]\ v^{LC}_{\xi_1} \otimes v^{LC}_{\xi_2}. \tag{54}$$

Again, Eq. (54) can be generalized to a frame where the nucleon has momentum P, which gives

$$|0,0\rangle^{(vv)} = \frac{m\sqrt{2}}{m+k^0} \sum_{\xi_1,\xi_2} \left[\bar{v}^{LC}_{\xi_1} \frac{\gamma \cdot P - M}{2M} \gamma_5 C (\bar{v}^{LC}_{\xi_2})^T\right]\ v^{LC}_{\xi_1} \otimes v^{LC}_{\xi_2}. \tag{55}$$

From Eq. 55 we can see that, by applying the Melosh transformation to the v^{inst} spinors, a new component of the Dirac representation is generated. All others are constructed similarly.

For completeness we mention three-quark states of spin $\frac{1}{2}$ and negative parity with a single v spinor that we expect to play a role in sea-quark Fock states

$$\begin{aligned}
vuu\ :\ |MA,\lambda\rangle^{(wuu)} &\equiv \sum_{\lambda_3,\lambda} C^{0\ \frac{1}{2}\ \frac{1}{2}}_{0\ \lambda_3\ \lambda} (\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\ w^{inst}_{\lambda_1} \otimes u^{inst}_{\lambda_2}) \otimes u^{inst}_{\lambda_3} \\
|MA,\lambda\rangle^{(uwu)} &\equiv \sum_{\lambda_3,\lambda} C^{0\ \frac{1}{2}\ \frac{1}{2}}_{0\ \lambda_3\ \lambda} (\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\ u^{inst}_{\lambda_1} \otimes w^{inst}_{\lambda_2}) \otimes u^{inst}_{\lambda_3} \\
|MA,\lambda\rangle^{(uuw)} &\equiv \sum_{\lambda_3,\lambda} C^{0\ \frac{1}{2}\ \frac{1}{2}}_{0\ \lambda_3\ \lambda} (\sum_{\lambda_1,\lambda_2} C^{\frac{1}{2}\ \frac{1}{2}\ 0}_{\lambda_1\ \lambda_2\ 0}\ u^{inst}_{\lambda_1} \otimes u^{inst}_{\lambda_2}) \otimes w^{inst}_{\lambda_3}.
\end{aligned} \tag{56}$$

There are also the corresponding mixed symmetric vuu spin invariants, which result from replacing the Clebsch-Gordan coefficients for pair spin 0 by those for pair spin 1.

A flux tube breaking in the nucleon, then, involves a quark pair creation event from the vacuum and generates an intermediate v-state, which may be converted by a valence gluon as in Fig.2a, or by a Goldstone boson in effective chiral field theory as in Fig.2b, into a u-spinor. Thus, these diagrams represent transition amplitudes from a three-quark Fock state to a three-quark-gluon or a three-quark-Goldstone boson Fock state mediated by quark pair creation. As suggested by chiral quark models, these Fock states are expected to contribute to the neutron charge form factor in particular, which relativistic quark model can not explain on the basis of the canonical (UUU) three-quark wave function alone. However, in scalar coupling (G_2 of Table I) the neutron charge form factor description improves [9]. The vuu states also enter and are probed by time-like weak processes discussed in ref. [6].

A key feature of the quark-nucleon coupling involving an antiquark is that the vertex function Λ is in general no longer directly related to the radial wave function. This point has

been emphasized in recent analyses [19,20] of skewed parton distributions of deeply virtual Compton scattering (DVCS) from the proton. When the leading twist handbag diagram of DVC is integrated over the longitudinal momentum fraction x, the integrand of the covariant triangle diagram results. In light-cone time-ordered perturbation theory the latter splits up into the standard form factor result involving the initial and final light-cone wave functions of the proton and the diagram shown in Fig. 3, where the photon momentum $q^+ > 0$ can no longer be chosen to vanish in DVCS. The antiquark-q^2-nucleon vertex function Λ involves $k^+ - q^+ < 0$. Using the Bethe-Salpeter equation for the proton projected to the null-plane, where it becomes the Weinberg equation, the negative momentum fraction may be shifted into the kernel V of the equation of motion, whereas the proton light-cone wave function ψ retains positive plus-momentum. In our case, using the $^3P_0$ pair creation amplitude $g_p \bar{v}_1 u_1$, the proton spin-flavor wave function $\bar{u}_2\gamma_5 C\bar{u}_3^T(\bar{u}_1 u_N) - \bar{u}_1\gamma_5 C\bar{u}_3^T(\bar{u}_2 u_N)$ is converted through the $u_1\bar{u}_1$ spin sum to the spin-flavor vertex structure

$$\bar{u}_2\gamma_5 C\bar{u}_3^T(\bar{v}_1(q-k)u_N) - \bar{v}_1(q-k)\gamma_5 C\bar{u}_3^T(\bar{u}_2 u_N),$$

where the v-spinor has the proper positive plus-component as a consequence of the pair creation from the vacuum. The vertex function $\Lambda \sim g_p R(M_0^2)(m_N^2 - M_0^2)$ remains connected to the radial light-cone momentum wave function R of the final proton state.

VI. DISCUSSION AND CONCLUSIONS

We have constructed relativistic three-quark states for the nucleon and several N*s in the Dirac representation and compared with the Bargmann-Wigner basis by systematically including all dynamically accessible matrix elements $(\bar{u}V)$, $(\bar{v}U)$, and $(\bar{v}V)$. Our direct and transparent construction of the Dirac basis states makes its equivalence with the Bargmann-Wigner basis manifest, thus avoiding the evaluation of many-body overlap matrix elements of ref. [8]. We have shown that in light front dynamics the nonstatic quark-baryon couplings (wave functions with zero static limit) can be constructed via unitary transformations from the instant form as well. Therefore, in a Lorentz covariant framework, they form part of a Hilbert space and should be treated on equal footing with the Melosh rotated nonrelativistic states. We also have compiled all linear relations among the symmetrized quark-nucleon couplings, as well as those for the N*(1520) and N*(1535) nucleon resonance states, from which the number of independent basis states follows. The Melosh transformations facilitate the construction of states with one or two antiquarks which are ingredients in transition amplitudes from three-quark to three-quark-gluon or three-quark-Goldstone boson Fock states.

VII. ACKNOWLEDGEMENT

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TABLES

$G_1 =$	$M(i\tau_2 C)_{12} \otimes (\gamma_5 u_\lambda)_3$
$G_2 =$	$M(i\tau_2\gamma_5 C)_{12} \otimes (u_\lambda)_3$
$G_3 =$	$M(\gamma^\mu \vec{\tau} i\tau_2 C)_{12} \otimes \vec{\tau}(\gamma_5\gamma_\mu u_\lambda)_3$
$G_4 =$	$M(i\tau_2\gamma^\mu\gamma_5 C)_{12} \otimes (\gamma_\mu u_\lambda)_3$
$G_5 =$	$\gamma \cdot P i\tau_2 \vec{\tau} C \otimes (\gamma_5 \vec{\tau} u_\lambda)_3$
$G_6 =$	$(i\tau_2 \gamma \cdot P \gamma_5 C)_{12} \otimes (u_\lambda)_3$
$G_7 =$	$M(\sigma^{\mu\nu} \vec{\tau} i\tau_2 C)_{12} \otimes (\gamma_5 \sigma_{\mu\nu} \vec{\tau} u_\lambda)_3$
$G_8 =$	$i(\sigma^{\mu\nu} P_\nu \vec{\tau} i\tau_2 C)_{12} \otimes (\vec{\tau}\gamma_5\gamma_\mu u_\lambda)_3$

Note that in each of the bases u_λ is understood as containing both the isospin and spin components of the nucleon. $C = i\gamma^2\gamma^0$ is the charge conjugation matrix.

TABLE I. **Spinor invariants of the nucleon in (12)3 coupling of the Dirac basis**

$G_1 =$	$M(UV+VU) \otimes (-\uparrow\downarrow + \downarrow\uparrow) \otimes V^\uparrow$
$G_2 =$	$M(UU+VV) \otimes (-\uparrow\downarrow + \downarrow\uparrow) \otimes U^\uparrow$
$G_3 =$	$-M[(UV-VU) \otimes (\uparrow\downarrow - \downarrow\uparrow) \otimes V^\uparrow - 2(UU-VV) \otimes (\uparrow\uparrow) \otimes U^\downarrow$ $+(UU-VV) \otimes (\uparrow\downarrow + \downarrow\uparrow) \otimes U^\uparrow]$
$G_4 =$	$-M[(UU-VV) \otimes (\uparrow\downarrow - \downarrow\uparrow) \otimes U^\uparrow - 2(UV-VU) \otimes (\uparrow\uparrow) \otimes V^\downarrow$ $+(UV-VU) \otimes (\uparrow\downarrow + \downarrow\uparrow) \otimes V^\uparrow]$
$G_5 =$	$-M(UV-VU) \otimes (\uparrow\downarrow - \downarrow\uparrow) \otimes V^\uparrow$
$G_6 =$	$-M(UU-VV) \otimes (\uparrow\downarrow - \downarrow\uparrow) \otimes U^\uparrow$
$G_7 =$	$-2M[(UU+VV) \otimes (\uparrow\downarrow + \downarrow\uparrow) \otimes U^\uparrow - 2(UU+VV) \otimes (\uparrow\uparrow) \otimes U^\downarrow$ $+(UV+VU) \otimes (\uparrow\downarrow + \downarrow\uparrow) \otimes V^\uparrow - 2(UV+VU) \otimes (\uparrow\uparrow) \otimes V^\downarrow]$
$G_8 =$	$-M[-2(UU+VV) \otimes (\uparrow\uparrow) \otimes U^\downarrow + (UU+VV) \otimes (\uparrow\downarrow + \downarrow\uparrow) \otimes U^\uparrow]$

TABLE II. **Conversion Table from DM Basis to BW Basis**

	J_S	J_V	J_T	J_A	J_P
J'_S	1/4	1/4	1/8	-1/4	1/4
J'_V	1	-1/2	0	-1/2	-1
J'_T	3	0	-1/2	0	3
J'_A	-1	-1/2	0	-1/2	1
J'_P	1/4	-1/4	1/8	1/4	1/4

Example: $J'_V = J_S - \frac{1}{2}J_V - \frac{1}{2}J_A - J_P$
Definitions:
$J_S = (\bar{u}_1 u_2)(\bar{u}_3 u_N)$
$J'_S = (\bar{u}_3 u_2)(\bar{u}_1 u_N)$
$J_V = (\bar{u}_1 \gamma_\mu u_2)(\bar{u}_3 \gamma^\mu u_N)$
$J'_V = (\bar{u}_3 \gamma_\mu u_2)(\bar{u}_1 \gamma^\mu u_N)$
$J_T = (\bar{u}_1 \sigma_{\mu\nu} u_2)(\bar{u}_3 \sigma^{\mu\nu} u_N)$
$J'_T = (\bar{u}_3 \sigma_{\mu\nu} u_2)(\bar{u}_1 \sigma^{\mu\nu} u_N)$
$J_A = (\bar{u}_1 \gamma_5 \gamma_\mu u_2)(\bar{u}_3 \gamma^5 \gamma^\mu u_N)$
$J'_A = (\bar{u}_3 \gamma_5 \gamma_\mu u_2)(\bar{u}_1 \gamma^5 \gamma^\mu u_N)$
$J_P = (\bar{u}_1 \gamma_5 u_2)(\bar{u}_3 \gamma^5 u_N)$
$J'_P = (\bar{u}_3 \gamma_5 u_2)(\bar{u}_1 \gamma^5 u_N)$

TABLE III. **Fierz Transformation Table**

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8
S_1	0	0	1/2	0	0	0	-1/4	0
S_2	0	0	-1/2	0	0	0	-1/4	0
S_3	0	0	-3	0	0	0	0	0
S_4	0	0	-1	0	0	0	0	0
S_5	0	0	-1/2	0	-1	0	-1/4	1
S_6	0	0	-1/2	0	1	0	1/4	-1
S_7	0	0	0	0	0	0	-3	0
S_8	0	0	-1/2	0	2	0	-1/4	-2

TABLE IV. **Symmetrized Spinor Invariants for** N

$G_1 =$	$M(\vec{\tau}i\tau_2 C)_{12} \otimes (\vec{\tau}\gamma_5(s_\mu \cdot U_\lambda^{\frac{1}{2},\mu}))_3$
$G_2 =$	$M(\vec{\tau}i\tau_2\gamma_5 C)_{12} \otimes (\vec{\tau}(s_\mu \cdot U_\lambda^{\frac{1}{2},\mu}))_3$
$G_3 =$	$M(\gamma^\mu i\tau_2 C)_{12} \otimes (\gamma_5\gamma_\mu(s_\mu \cdot U_\lambda^{\frac{1}{2},\mu}))_3$
$G_4 =$	$M(\vec{\tau}i\tau_2\gamma^\mu\gamma^5 C)_{12} \otimes (\vec{\tau}\gamma_\mu(s_\mu \cdot U_\lambda^{\frac{1}{2},\mu}))_3$
$G_5 =$	$(\gamma \cdot P i\tau_2 C)_{12} \otimes (\gamma_5(s_\mu \cdot U_\lambda^{\frac{1}{2},\mu}))_3$
$G_6 =$	$(\vec{\tau}i\tau_2\gamma \cdot P\gamma_5 C)_{12} \otimes (\vec{\tau}(s_\mu \cdot U_\lambda^{\frac{1}{2},\mu}))_3$
$G_7 =$	$M(\sigma^{\mu\nu}i\tau_2 C)_{12} \otimes (\gamma_5\sigma_{\mu\nu}(s_\mu \cdot U_\lambda^{\frac{1}{2},\mu}))_3$
$G_8 =$	$(i\sigma^{\mu\nu}P_\nu i\tau_2 C)_{12} \otimes (\gamma_5\gamma_\mu(s_\mu \cdot U_\lambda^{\frac{1}{2},\mu}))_3$
Note: $U_\lambda^{\frac{1}{2},\mu} = C^{1\ \frac{1}{2}\ \frac{1}{2}}_{m_1 m_2 \lambda}\hat{\epsilon}_{m_1} u_{m_2} = -\sqrt{\frac{1}{3}}\ \hat{\epsilon}_0^\mu u_{\frac{1}{2}} + \sqrt{\frac{2}{3}}\hat{\epsilon}_{+1}^\mu u_{-\frac{1}{2}}$	

TABLE V. **Spinor Invariants for $N^*(1535)$ in (12)3 coupling of the Dirac basis**

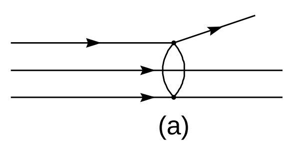
$G_1 =$	$M(\vec{\tau}i\tau_2 C)_{12} \otimes (\vec{\tau}\gamma_5(s_\mu \cdot U_\lambda^{\frac{3}{2},\mu})_3)$
$G_2 =$	$M(\vec{\tau}i\tau_2\gamma_5 C)_{12} \otimes (\vec{\tau}(s_\mu \cdot U_\lambda^{\frac{3}{2},\mu}))_3$
$G_3 =$	$M(\gamma^\mu i\tau_2 C)_{12} \otimes (u_3\gamma_5\gamma_\mu(s_\mu \cdot U_\lambda^{\frac{3}{2},\mu}))_3$
$G_4 =$	$M(\vec{\tau}i\tau_2\gamma^\mu\gamma^5 C)_{12} \otimes (\vec{\tau}\gamma_\mu(s_\mu \cdot U_\lambda^{\frac{3}{2},\mu}))_3$
$G_5 =$	$(\gamma \cdot P i\tau_2 C\bar{u}_2^T)_{12} \otimes (\gamma_5(s_\mu \cdot U_\lambda^{\frac{3}{2},\mu}))_3$
$G_6 =$	$(\vec{\tau}i\tau_2\gamma \cdot P\gamma_5 C)_{12} \otimes (\tau(s_\mu \cdot U_\lambda^{\frac{3}{2},\mu}))_3$
$G_7 =$	$M(\sigma^{\mu\nu}i\tau_2 C)_{12} \otimes (\gamma_5\sigma_{\mu\nu}(s_\mu \cdot U_\lambda^{\frac{3}{2},\mu}))_3$
$G_8 =$	$i(\sigma^{\mu\nu}P_\nu i\tau_2 C)_{12} \otimes (\gamma_5\gamma_\mu(s_\mu \cdot U_\lambda^{\frac{3}{2},\mu}))_3$
Note: $U_\lambda^{\frac{3}{2},\mu} = C^{1\ \frac{1}{2}\ \frac{3}{2}}_{m_1 m_2 \lambda}\hat{\epsilon}_{m_1} u_{m_2}, \quad U_{\frac{3}{2}}^{\frac{3}{2},\mu} = \hat{\epsilon}_1 u_{\frac{1}{2}}, \quad U_{\frac{1}{2}}^{\frac{3}{2},\mu} = \sqrt{\frac{2}{3}}\hat{\epsilon}_0^\mu u_{\frac{1}{2}} + \sqrt{\frac{1}{3}}\hat{\epsilon}_{+1}^\mu u_{-\frac{1}{2}}.$	

TABLE VI. **Spinor Invariants for $N^*(1520)$ in (12)3 coupling of the Dirac basis**

	G_1	G_2	G_3^*	G_4	G_5^*	G_6	G_7^*	G_8
S_1	-7/4	1/4	1/4	-1/4	0	0	1/8	0
S_2	1/4	-7/4	-1/4	1/4	0	0	1/8	0
S_3	-1	1	1/2	1/2	0	0	0	0
S_4	-1	1	-1/2	-5/2	0	0	0	0
S_5	1/4	-1/4	-1/4	-1/4	-1/2	1/2	1/8	1/2
S_6	-1/4	1/4	-1/4	-1/4	-1/2	-3/2	-1/8	-1/2
S_7	-3	-3	0	0	0	0	1/2	0
S_8	3/4	3/4	1/4	-1/4	1	1	-1/8	0
$G_i \sim s_{3,\mu} \cdot U_{\frac{1}{2}}^{\frac{1}{2},\mu}$, where $s_3 = p_1 - p_2$								
$G_i^* \sim s_{m,\mu} \cdot U_{\frac{1}{2}}^{\frac{1}{2},\mu}$, where $s_m = 2p_3 - p_1 - p_2 = s_2 - s_1$								

TABLE VII. **Symmetrized Spinor Invariants for $N^*(1535)$ in DM Basis**

FIGURES



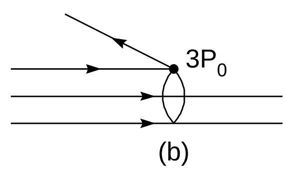


FIG. 1. (a) Three-quark-nucleon vertex: uuu; (b) uuv via quark pair creation

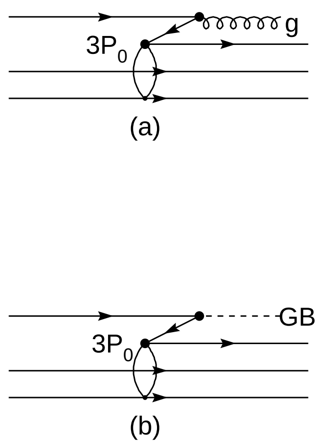


FIG. 2. (a) Three-quark to three-quark-gluon Fock state transition amplitude; (b) three-quark to three-quark-Goldstone boson Fock state transition amplitude

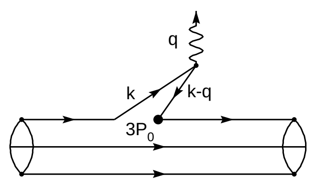


FIG. 3. (a) Triangle Z-diagram

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